**Conversions**

1. **Rapid conversions**
2. **Substitution method**
3. **Successive divisions and multiplication**

1. **Rapid conversions:**conversions between bases which are powers of2.

* 1. **Conversion from the source base p=2k, p{4=22,8=23,16=24} into the destination base  2**

**Rule**:

**Each digit from the source number in base *p*=2*k*, the integer part and the fractional one, will be replaced by the corresponding group of *k* binary digits (adding if it is necessary insignificant zeros to the left).**

* 1. **. Conversion from base 2 into the destination base q=2k** , **q{4=22,8=23,16=24}**

**Rules:**

* **for the *integer/fractional  part*: from *right/left* to *left/right* (relative to the decimal point) make groups of *k*binary digits (eventually we add to the *left/right* insignificant zeros to have a complete group);**
* **the groups will be replaced by the corresponding digits in  base *q*=2*k.***

**Example 1:**

**(8) =      ? (2)       =  ? (16)**

**5327,321(8)=101 011 010 111,011 010 001(2)=**

**1010 1101 0111,0110 1000 1000(2)=AD7,688(16)**

**5(8)=5=4+1=2^2+2^0=        101 (2)**

**D(16)=13=2^3+2^2+2^0=1101(2)**

**115=64+32+16+2+1=2^6+2^5+2^4+2^1+2^0=1110011(2)**

**Example 2:**

**03 32 10,32 30  (4)  = 3E4,EC  (16)**

**32(4)=2 \* 4^0 + 3\*4^1 = 2 + 12 = 14 = E(16)**

**Example 3:**

**6BEA , BEC    (16) =  12 23 32 22, 23 32 30 (4)**

**B(16)=  11 = 2\*4^1 + 3 \* 4^0 = 23(4)**

**E(16)= 14 = 3\*4 + 2 = 32(4)**

1. **Substitution method**

* **calculations in the destination base**
* **it is recommended for *b < h*, *b* (source base),  *h*(destination base)**

***Steps:***

* **all the digits from the source representation are converted into the destination base:**
* **the base*b* is converted into base *h*:**
* **we calculate in base *h* the sum of the positional values**

**Example 4:with a precision of 2 digits at the fractional part in the destination representation**

**1432,31(5)=  ?      ,    (8)**

**1(5) = 1(8),  4(5) = 4(8), 3(5) = 3(8), 2(5) = 2(8), 5=5(8)**

**1432,31(5) = 1(8) \* 5(8) ^ 3 + 4(8) \* 5(8)^2 + 3(8) \* 5(8) ^ 1 + 2(8) \* 5(8) ^ 0 +**

**+  3(8) \* 5(8)^(-1) + 1 \* 5(8)^(-2)= 175 (8)+144(8)+17(8)+2(8)+ 0,46(8)++0,02(8) =**

**= 362,50(8)**

**!!! Calculations in base 8!!!**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **c** | **3** | **0** |  |  | **1** | **0** | **0** |  |  |  | 1 | 0 | **0** |  |  |  | **0** |  |  |  |  |  |  |  |  |  |  |  |
|  |  | **5** | **\*** |  |  | **3** | **1** | **\*** |  |  |  | **3** | **1** | **\*** |  |  | **3** | **\*** |  |  |  |  |  |  |  |  |  |  |
|  |  | **5** |  |  |  |  | **5** |  |  |  |  |  | **4** |  |  |  | **5** |  |  |  |  |  |  |  |  |  |  |  |
|  | **3** | **1** |  |  | **1** | **7** | **5** |  |  |  | **1** | **4** | **4** |  |  | **1** | **7** |  |  |  |  |  |  |  |  |  |  |  |

**5(8)^2=31(8)**

**5\*5=25, 25 mod 8 = 1, carry = 3**

**5(8)^3=  175(8)**

**3\*5 = 15, 15 mod 8 = 7, carry = 1**

**4\*3=12,12 mod 8 =4 , carry 1**

**3(8)\*5(8)=17(8)**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **3,00 (8)** | **: 5(8)** | **1,00(8)** | **5(8)** | **0,14(8)** | **5(8)** |  |
| **/** | **0,46 (8)** | **/** | **0,14** | **/** | **0,02** |
| **30** | | **10** | | **1** | |
| **/** | | **/** | | **/** | |
| **40** | | **30** | | **14** | |
| **2** | | **4** | | **2** | |

           30(8)=3 \* 8 + 0 = 24, 24 div 5 = 4, 24 mod 5 = 4

40(8) = 4 \* 8 + 0 = 32, 32 div 5 = 6, 32 mod 5 = 2

**10(8) = 1 \* 8 = 8 = 8, 8 div 5 =1, 8 mod 5 = 3**

**30(8) = 3 \* 8 = 24, 24 div 5 = 4, 24 mod 5 = 4**

**14(8) = 8 + 4 = 12, 12 div 5 = 2, 12 mod 5 = 2**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **c** | **1** | **2** | **0** |  | **1** | **0** |  |
|  | **1** | **7** | **5** | **,** | **0** | **0** |  |
|  | **1** | **4** | **4** | **,** | **0** | **0** |  |
|  |  | **1** | **7** | **,** | **0** | **0** |  |
|  |  |  | **2** | **,** | **0** | **0** |  |
|  |  |  | **0** | **,** | **4** | **6** |  |
|  |  |  | **0** | **,** | **0** | **2** |  |
|  | **3** | **6** | **2** | **,** | **5** | **0** |  |

**0+5+4+7+2+0+0 = 18 (10)   18 div 8 = 2 , 18 mod 8 = 2**

**1432,31(5) = 362,50(8)**

**Example 5:with a precision of 2 digits at the fractional part in the destination representation**

 !!!calculations in base 16

**1452, 36 (7) = ? (16)**

 1452,36(7) = 1(16)\*7(16)^3 + 4(16)\*7(16)^2 + 5(16)\*7(16)^1 + 2(16)\*7(16)^0 + 3(16)\*7(16)^(-1) + 6(16)\*7(16)^(-2) = 157(16)+C4(16)+23(16)+2(16)+0,6D(16)+ 0,1F(16) =

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| c | 3 | 0 |  | 1 | 0 | 0 |  |  |  | 0 | 0 |  | 2 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | 7 | \* |  | 3 | 1 | \* |  |  | 3 | 1 | \* |  | 5 | \* |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | 7 |  |  |  | 7 |  |  |  |  | 4 |  |  | 7 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 3 | 1 |  | 1 | 5 | 7 |  |  |  | C | 4 |  | 2 | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

  7(16)^2= 31(16)

  7(16)^3=31(16)\*7(16)=157(16)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **3,00(16)** | :7(16) | 6,00(16) | :7(16) | 0,DB(16) | :7(16) |  |  |
| 30 | 0,6D | 60 | 0,DB | 0D | 0,1F |  |  |
| 60 | | 50 | | 6B | |  | |
| 5 | | 3 | |  | |  | |
|  | |  | |  | |  | |
|  | |  | |  | |  | |
|  | |  | |  | |  | |

  30(16)=3\*16+0=48, 48 mod 7=6, 48 div 7=6

60(16)=6\*16+0=96, 96 mod 7=5, 96 div 7=13😃(16)

  50(16)=5\*16+0=80, 80 mod 7=3, 80 div 7=11=B(16)

0D(16) = 13, 13 mod 7 = 6, 13 div 7 = 1

6B(16) = 6\*16 + 11 = 96 + 11 = 107 mod 7 =2, 107 div 7 = 15=F(16)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **c** | 1 | 1 | 0 |  | 1 |  | +(16) |
|  | 1 | 5 | 7 | , | 0 | 0 |  |
|  |  | C | 4 |  | 0 | 0 |  |
|  |  | 2 | 3 | , | 0 | 0 |  |
|  |  |  | 0 | , | 6 | D |  |
|  |  |  | 0 | , | 1 | F |  |
|  |  |  | 2 | , | 0 | 0 |  |
|  | 2 | 4 | 0 | , | 8 | C |  |

**1452, 36 (7) = 240,  8C** **(16)**

1. **The method of successive divisions/multiplications**

* **it is recommended for *h < b*, *b* –source base and *h*- destination base.**
* **calculations in the source base**

**Integer part**: **successive divisions** by the destination base (***h***) are performed

* the process of successive divisions ends when 0 is obtained as quotient.
* the remainders, in the reverse order, are the digits of the new representation in base q*.*

**Fractional part**: **successive multiplications**by the destination base (***h***) are performed

* the fractional part is multiplied by ***b*** obtaining a number with an integer part and a fractional one;
* we continue with the multiplication of this new fractional part,...
* the process of the successive multiplications continues until one of the following conditions is satisfied:

          a)  the fractional part becomes 0;

          b) an established number of digits of the fractional part were calculated;

          c) periodicity is obtained.

* the integer parts, in the order of obtaining them during the multiplications process, are the digits of the fractional part in the destination representation.

**Example 6: with a precision of 3 digits at the fractional part in the destination representation**

 !!calculations in base 8

**362,50(8)= ?**  **, (5)**

**Conversion of the integer part**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **362 (8)** | :5(8) | 60(8) | :5(8) | 11(8 | ) | :5(8) | 1(8) | :5(8) |
| 36 | 60 | 10 | 11 | 4 |  | 1 | 1 | 0 |
| 02 | | 3 | |  |  | |  | |
| 2 | |  | |  |  | |  | |
|  | |  | |  |  | |  | |
|  | |  | |  |  | |  | |
|  | |  | |  |  | |  | |

 36(8) = 6 + 3 \* 8 = 6 + 24 = 30, 30 / 5 = 6, 30 % 5 = 0

10(8) = 8, 8 / 5 = 1, 8 % 5 = 3

11(8) = 1 + 8 = 9, 9 / 5 = 1, 9 % 5 = 4

**Calculations:**

**Conversion of the fractional part** : 0,5(8)=0,(30) (5)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| c | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | **0,** | 5 |  | **\*** |  |  |  |  | **\*** |  |  |  |  | **\*** |  |
|  |  | 5 |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 3, | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |

0,5(8)\*5(8)= 3,1(8)

 25 div 8 = 3 , 25 mod 8 = 1

 0,1(8) \* 5(8)=0,5 (8)

**362,50(8)= ~**1432 **,(30) (5)**

**Example 7: with a precision of 3 digits at the fractional part in the destination representation**

 !!! Calculations in base 16

**240,  8C  (16) = ?  ,  (7)**

**Conversion of the integer part** :

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **240(16)** | :7(16) | 52(16) | :7(16) | B(16) | :7(16) | 1(16) | :7(16) |
| 24 | 52 | 5 | B | 4 | 1 | 1 | 0 |
| 10 | |  | |  | |  | |
| 2 | |  | |  | |  | |
|  | |  | |  | |  | |
|  | |  | |  | |  | |

24(16)= 4+2\*16=4+32=36, 36 mod 7 = 1, 36 div 7 = 5

10(16) = 16 , 16 mod 7 = 2 , 16 div 7 = 2

52(16) = 2+5\*16 = 2 + 80 = 82 , 82 mod 7 = 5 , 82 div 7 = 11 = B(16)

**Conversion of the fractional part**  0,8C(16)=0, 35 (7)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| c | 3 | 5 | 0 |  |  | 55 | 1 | 0 |  |  |  |  |  |  |  |
|  | **0,** | 8 | C | **\*** |  | 0, | D | 4 | **\*** |  |  |  |  | **\*** |  |
|  |  |  | 7 |  |  |  |  | 7 |  |  |  |  |  |  |  |
|  | 3, | D | 4 |  |  | 5,5, | CC | C |  |  |  |  |  |  |  |

0,8C(16)\* 7(16)= 3,D4(16)

  C(16) \* 7(16) = 12\*7 =84 , 84/16 = 5 , 84 mod 16=4

8 \*7 +5 = 61, 61 / 16 = 3, 61%16 = 13 = D(16)

0,D4(16)\*7(16)=5,CC(16)

4\*7 =28, 28 /16 = 1, 28%16 = 12 =C(16)

D(16)\*7(16)+1(16) = 13\*7 +1 = 92,92 / 16 = 5, %16 = 12 = C(16) 92%16=C(16)

**240,  8C  (16) =~    1452  , 35  (7)**